

26/10/2015

الاسم

م. خطاب

الصفحة [4]

Inverse Z-transform

$$Y(z) \xrightarrow{z^{-1}} y(n)$$

① Long Division (open form).

② Partial fraction (Closed form).

Ex: Given $Y(z) = \frac{z(z+1)}{(z-0.3)(z-0.4)(z-1)^2}$

Request = $y(0), y(1), \dots, y(4)$

Using ① Long division

② Partial fraction

Solution

$$Y(z) = \frac{z^2 + z}{z^4 - 2.7z^3 + 2.52z^2 - 0.82z + 0.12}$$

$$\begin{array}{r} z^2 + 3.7z^3 + 7.48z^4 \\ \hline z^4 - 2.7z^3 + 2.52z^2 + 0.82z + 0.12 \\ \hline \end{array}$$

$$\begin{array}{r} z^2 + z \\ \hline (z^2 - 2.7z + 2.52 + 0.82z^{-1} + 0.12z^{-2}) \\ \hline 3.7z - 2.52 + 0.82z^{-1} + 0.12z^{-2} \\ \hline (3.7z - 10 + 9.32z^{-1} - 3.03z^{-2} + 0.44z^{-3}) \\ \hline 7.48 - 8.5z^{-1} + 2.91z^{-2} - 0.44z^{-3} \end{array}$$

$$Y(z) = z^{-2} + 3.7z^{-3} + 7.48z^{-4} + \dots$$

$$Y(z) = \sum_{k=0}^{\infty} y(k) z^{-k}$$

$$\Rightarrow y(0) = 0 \quad ; \quad y(1) = 0 \quad ; \quad y(2) = 1 \quad ;$$

$$y(3) = 3.7 \quad ; \quad y(4) = 7.48$$

$$Y(z) = \frac{z(z+1)}{(z-0.4)(z-0.3)(z-1)^2} \Rightarrow \text{obtain partial fraction}$$

$$= z \left[\frac{-26.53}{z-0.3} + \frac{38.88}{z-0.4} + \frac{4.76}{(z-1)^2} + \frac{-12.34}{z-1} \right]$$

$$y(n) = -26.53(0.3)^n + 38.88(0.4)^n + 4.76n - 12.34$$

$$n=0 \Rightarrow y(0)=0; y(1)=0 \dots$$

$$x_1(n) = 5\delta(n) - 2\delta(n-2)$$

$$x_2(n) = 3\delta(n-2)$$

Find $y(n) = x_1(n) * x_2(n)$

① Linear Convolution

② inverse Z transform

Report